BOLINAS-STINSON UNION SCHOOL DISTRICT
125 Olema-Bolinas Road | Main Office (415) 868-1603
Bolinas, CA 94924 | Fax (415) 868-9406
http://bolinas-stinson.org | Stinson Campus (415) 868-0844

PRIMARY GRADES MATHEMATICS
STANDARDS FOR MATHEMATICAL PRACTICE, ESSENTIAL STANDARDS, STUDENT WORK HABITS, AND VOCABULARY
JUNE 2010

Bolinas-Stinson Union School District teachers seek to produce proficient math students as part of our balanced and enriched full academic program. Proficient students expect mathematics to make sense. They take an active stance in solving mathematical problems. When faced with a non-routine problem, they have the courage to plunge in and try something, and they have the procedural and conceptual tools to carry through. They are experimenters and inventors, and can adapt known strategies to new problems. They think strategically.

Students who engage in these practices discover ideas and gain insights that spur them to pursue mathematics beyond the classroom walls. They learn that effort counts in mathematical achievement. These are practices that expert mathematical thinkers encourage in apprentices. Encouraging these practices in our students is as much a goal of the mathematics curriculum as is teaching specific content topics and procedures. Taken together with the standards for mathematical content, they support eventual productive entry into high school and college courses or career pathways.[1]

The following list of essential standards in mathematics is comprised of content topics and procedures that the Bolinas-Stinson Union School District faculty judge to be guaranteed essential learnings at each grade level. The standards are taken from the California Department of Education Content Standards and Framework in Mathematics. The skills, knowledge, and Student Work Habits identified in this document do not comprise the whole of mathematics instruction at each grade level, but identify only the *essential* learnings students need in order to be successful in a Kindergarten through grade 12 sequence of instruction in mathematics. In practice, Bolinas-Stinson School teachers offer a broader mathematics curriculum that includes standards not detailed in this document. However, teachers are expected carefully to assess student mastery of these essential learnings and offer additional instruction to students who have not learned them.

The essential standards are listed for the following strands of mathematics instruction: Number, Algebra and Functions, Geometry, and Data Analysis, Statistics, and Probability. In addition, students are expected to learn and practice the listed "Student Work Habits". Also listed here are essential mathematics vocabulary to be learned and practiced. However, these elements by themselves fall short of a high-quality mathematics program. In addition to the content standards, vocabulary, and work habits, teachers will guide students to engage in the Standards for Mathematical Practice.

The Standards for Mathematical Practice form the background of all mathematics instruction. They are explained in the Common Core State Standards in Mathematics developed in 2010 and adopted by the California Department of Education. The Common Core State Standards Initiative is a state-led effort

1 From the Common Core Standards Initiative: http://www.corestandards.org/

coordinated by the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO).

These are the K-12 Standards for Mathematical Practice Bolinas-Stinson School teachers will cultivate in K-8 students.[2]

Primary grades students engage in these practices at a developmentally appropriate level which lays the groundwork for later sophistication of practice and thought through the upper grades. A hands-on approach to learning mathematics is absolutely essential to develop students' understanding. Students will make regular use of objects to help them learn.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education, including the processes of problem solving, reasoning and proof, communication, representation, making connections, and proficiencies including adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy). ***Bolinas-Stinson School teachers engage with students in a variety of ways at their appropriate developmental levels in order to develop, over a 9-year progression, these mathematical practices.***

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

[2] Adapted from the Common Core State Standards Initiative: http://www.corestandards.org/the-standards/mathematics/introduction/standards-for-mathematical-practice/

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models and objects, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use a variety of tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the essential content standards

The Standards for Mathematical Practice describe ways in which developing student practitioners of the

discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years.

Mathematics Content Standards should be a balanced combination of procedure and understanding. The simple expectation of students that they "understand" in addition to merely performing procedures presents especially good opportunities to connect the 8 practices outlined above to the essential content standards outlined below. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use a variety of tools mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

Primary Grades Mathematics: Essential Standards Map (June 2010)

This is a living document. Bolinas-Stinson Union School District faculty may reexamine and revise this document as necessary.

Kindergarten prerequisites:

Number Sense

- Count ten or more objects (touch and move each object as it is counted)

Geometry and Measurement

Algebra and Functions

- Patterns: Recognize an "ABAB" pattern (repeating pattern)
- Sort and group objects in a non-random fashion (big and small, different kinds of animals)

Statistics, Data Analysis, and Probability

Student Work Habits

- Use math manipulatives appropriately (tiles, blocks, cubes, Rekenreks, etc.)

Kindergarten Essentials:

Number Sense

- Count up to thirty with one-to-one correspondence.
- Read, write, name, and count numbers up to thirty.
- Model addition by counting two piles of objects, putting them together, and counting up again.
- Model subtraction by counting one pile of objects, taking a defined number away, and counting the remaining objects.

Geometry and Measurement

- Identify common two and three dimensional shapes.

Algebra and Functions
- Reproduce repeating (e.g.: ABAB; ABBABB;…) patterns by referring to shapes, sizes, or colors.

Statistics, Data Analysis, and Probability
- Reproduce repeating (e.g.: ABAB; ABBABB;…) patterns by referring to shapes, sizes, or colors.

Student work habits
- Be on task
- Work independently on investigation/practices
- Work constructively in small cooperative groups
- Attempt tasks that are required by teachers
- Use math manipulatives to assist with learning
- Listen attentively during demonstration

Grade 1 Essentials:

Number Sense
- Count, read, and write whole numbers to 100
- Accurately build any 2-digit number using correct tens and ones Dienne's blocks
- Solve up to two-digit addition and subtraction problems using concrete objects
- Count by 2's to 30, 5's, and 10's to 100.
- Count by 10's starting with numbers other than 10.
- Use the 100 chart to determine one more, one less, ten more, and ten less than a number between 10-100.
- Use the addition strategy of counting on, beginning with highest number in the equation (conservation of number)

Geometry and Measurement
- Measure objects by direct comparison with other objects of standard and non-standard units (Cuisinaire rods, arm lengths, etc.)

Algebra and Functions
- Describe and extend simple repeating patterns in a variety of contexts (rhythm, number, color, shape, etc.)

Statistics, Data Analysis, and Probability
- Describe and extend simple repeating patterns in a variety of contexts (rhythm, number, color, shape, etc.)
- Generate mathematical statements about bar graphs and picture graphs.

Student work habits
- Be on task and productive
- Work independently on investigation/practices
- Work constructively in small cooperative groups
- Persevere through a task
- Show thinking and work on written papers (with numbers, pictures, words)
- Use math manipulatives in an organized and purposeful way

- Listen attentively during demonstration

Grade 2 Essentials:

Number Sense
- Demonstrate understanding of whole number place value to one thousands place
- Skip count by 2's, 5's, and 10's
- Solve up to three-digit addition problems using expanded notation
- Add and subtract using a # line
- Know addition and subtraction facts (sums up to 20): Otter creek math addition and subtraction A-Z
- Recognize fractions as part of a group and as part of a whole
- Write numbers 0-20 as words

Geometry and Measurement
- Describe and classify two and three dimensional geometric shapes by mathematical attributes (e.g.: faces, edges, sides, vertices)

Algebra and Functions
- Demonstrate understanding of growth patterns and extend them in a variety of contexts (e.g.: number of ears on 1 horse, 2 horses, …)

Statistics, Data Analysis, and Probability
- Organize and interpret numerical data in pictographs, bar graphs, and Venn diagrams
- Know the names and order of the months of the year.

Student work habits
- Be on task and productive
- Use multiple problem solving strategies
- Work independently on investigation/practices
- Work constructively in small cooperative groups
- Persevere through a task
- Show thinking and work on written papers (with numbers, pictures, words)
- Use math manipulatives in an organized and purposeful way
- Listen attentively during demonstration

Mathematics Vocabulary

Language is a very powerful tool and should be used to foster the learning of mathematics. Communicating about mathematical ideas is a way for students to articulate, clarify, organize, and consolidate their thinking. Communication makes mathematical thinking observable and thereforefacilitates further development of that thought. It encourages students to reflect on their own knowledge and their own ways of solving problems. Communication can consist not only of conversations between student and teacher or one student and another student but also of students listening to a number of peers and joining group discussions in order to clarify, question, and extend conjectures. The discourse should not be a goal in itself but rather should be focused on making sense of mathematical ideas and using them effectively in modeling solving problems.

From Principles and Standards for School Mathematics, **NCTM**

Each grade level vocabulary list is inclusive of all previous grade levels and should be utilized in context. This is not a finite vocabulary list but rather an ongoing, expanding file.

Kindergarten Math Vocabulary

NUMBER WORDS

addition
eight
eighth
fifth
first
five
four
fourth
half
less than
more than
nine
ninth
one
quantities
same as
second
seven
seventh
six
sixth
subtraction
ten
tenth
third
three
two
whole
zero

GEOMETRY

above
angle
attribute
below
between
bottom
circle
curved
edge
face
hexagon
inside
middle
outside
oval
over
rectangle
side
square
three dimensional
straight
top
trapezoid
two dimensional figure
triangle
under

MEASUREMENT

area
calendar
clock
colder
coldest
day
equal
fewer
greater than
heavier
heaviest
height
hotter
hottest
length
less than
lighter
lightest
longer
longest
mass
month
more than
same
shorter
shortest
taller
tallest
temperature
week
weight
year

PROCESSESS AND TOOLS

answer
collect
combine
compare
connect
construct
count
create
describe
determine
develop
evaluate
extend
joining
model
name
observation
order
place
predict
reasonable
represent
separating
solve
strategy
symbol
understand
use

PROBABILITY & STATISTICS

graph

ALGEBRAIC THINKING

after
before
group
next
pattern
repeating pattern
sequence
sort

First Grade Math Vocabulary

NUMBER & OPERATION for 1st Grade

addition
first
half
less than
more than
quantities
same as
second
subtraction

***New for this grade level
amount
cent
difference
dime
doubles
equal groups
fewer
greater than
less than
minus
nickel
ones
place value
quarter
sum
total

GEOMETRY for 1st Grade

angle
attribute
edge
face
hexagon
oval
rectangle
rhombus
square
three dimensional
trapezoid
two dimensional figure
triangle
vertices

***New for this grade level
cone
prism
pyramid
rectangle
sphere

MEASUREMENT for 1st Grade

area
calendar
capacity
equal
fewer
greater than
heavier
heaviest
height
hotter
hottest
length
less than
lighter
lightest
longer
longest
mass
month
more than
same
shorter
shortest
taller
tallest
temperature
week
weight
year

***New for this grade level
clock
hour
minute
temperature

PROCESSES AND TOOLS for first grade

compare
connect
describe
extend
identify
model
observation
order
place
predict
reasonable
represent
separating
solve
symbol

***New for this grade level
collect
combine
conclusion
estimate
organize
reasonable
recognize
select
graph

PROBABILITY AND STATISTICS for 1st grade

graph
***New for this grade level
bar graph
certain
data
picture graph
table
tally mark

ALGEBRAIC THINKING for 1st grade

pattern
sequence
sort

***New for this grade level
additive pattern
even number
fact families
number sentence
odd number
prediction
repeating pattern
sets
skip count

Second Grade Math Vocabulary

NUMBER & OPERATION for 2nd Grade

addition
amount
difference
dime
doubles
equal groups
fact family
fewer
greater than
less than
minus
more than
penny
place value
quantities
quarter
subtraction
sum
total
whole

***New for this grade level
addend
difference
digit
divide
division
equivalent
multiplication
quantity
share equally
sum
total
whole
zero

GEOMETRY for 2nd Grade

angle
attribute
circle
cone
curved
edge
face
hexagon
oval
parallelogram
prism
pyramid
rectangle
rhombus
sphere
square
three dimensional
trapezoid
two dimensional figure
triangle
vertices

***New for this grade level
cylinder
hexagon
pentagon
polygon

MEASUREMENT for 2nd Grade

area
equal
fewer
height
length
minute
same

***New for this grade level
approximately
centimeter
foot
inch
length
meter
thermometer
yard

PROCESSESS AND TOOLS for 2nd Grade

answer
create
collect
combine
count
develop
estimate
evaluate
identify
order
organize
place
predict
solve
strategy
represent
symbol
systematic
understand

***New for this grade level
apply
generate
recall

PROBABILITY & STATISTICS for 2nd Grade

bar graph
data
graph
picture graph
table
tally mark

***New for this grade level
less likely
more likely
survey