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MIDDLE SCHOOL MATHEMATICS

STANDARDS FOR MATHEMATICAL PRACTICE, ESSENTIAL STANDARDS, STUDENT WORK HABITS, AND VOCABULARY

JUNE 2010

Bolinas-Stinson Union School District teachers seek to produce proficient math students as part of our balanced and enriched full academic program. Proficient students expect mathematics to make sense. They take an active stance in solving mathematical problems. When faced with a non-routine problem, they have the courage to plunge in and try something, and they have the procedural and conceptual tools to carry through. They are experimenters and inventors, and can adapt known strategies to new problems. They think strategically.

Students who engage in these practices discover ideas and gain insights that spur them to pursue mathematics beyond the classroom walls. They learn that effort counts in mathematical achievement. These are practices that expert mathematical thinkers encourage in apprentices. Encouraging these practices in our students is as much a goal of the mathematics curriculum as is teaching specific content topics and procedures. Taken together with the standards for mathematical content, they support eventual productive entry into high school and college courses or career pathways.[1]

The following list of essential standards in mathematics is comprised of content topics and procedures that the Bolinas-Stinson Union School District faculty judge to be guaranteed essential learnings at each grade level. The standards are taken from the California Department of Education Content Standards and Framework in Mathematics. The skills, knowledge, and Student Work Habits identified in this document do not comprise the whole of mathematics instruction at each grade level, but identify only the *essential* learnings students need in order to be successful in a Kindergarten through grade 12 sequence of instruction in mathematics. In practice, Bolinas-Stinson School teachers offer a broader mathematics curriculum that includes standards not detailed in this document. However, teachers are expected carefully to assess student mastery of these essential learnings and offer additional instruction to students who have not learned them.

The essential standards are listed for the following strands of mathematics instruction: Number, Algebra and Functions, Geometry, and Data Analysis, Statistics, and Probability. In addition, students are expected to learn and practice the listed "Student Work Habits". Also listed here are essential mathematics vocabulary to be learned and practiced. However, these elements by themselves fall short of a high-quality mathematics program. In addition to the content standards, vocabulary, and work habits, teachers will guide students to engage in the Standards for Mathematical Practice.

The Standards for Mathematical Practice form the background of all mathematics instruction. They are explained in the Common Core State Standards in Mathematics developed in 2010 and adopted by the California Department of Education. The Common Core State Standards Initiative is a state-led effort

1 From the Common Core Standards Initiative: http://www.corestandards.org/

coordinated by the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO).

These are the Standards for Mathematical Practice Bolinas-Stinson School teachers will cultivate in K-8 students:[2]

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education, including the processes of problem solving, reasoning and proof, communication, representation, making connections, and proficiencies including adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy). Bolinas-Stinson School teachers engage with students in a variety of ways at their appropriate developmental levels in order to develop, over a 9-year progression, these mathematical practices.

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they

[2] Adapted from the Common Core State Standards Initiative: http://www.corestandards.org/the-standards/mathematics/introduction/standards-for-mathematical-practice/

make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models and objects, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use a variety of tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude

they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the essential content standards

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years.

Mathematics Content Standards should be a balanced combination of procedure and understanding. The simple expectation of students that they "understand" in addition to merely performing procedures presents especially good opportunities to connect the 8 practices outlined above to the essential content standards outlined below. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use a variety of tools mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

Middle School Mathematics: Essential Standards Map (June 2010)

Bolinas-Stinson Union School District faculty may reexamine and revise this document as necessary.

Course I prerequisites:

Number Sense

- Understand place value to one billions and to one-hundred thousandths
- Add, subtract:
    - fractions and mixed numbers with unlike denominators
    - decimals
    - integers (positive and negative)
- Multiply and divide:
    - Simple related fractions (1/2 ÷ ¼; ½ x 2/3)
    - Decimals
    - Large whole numbers
- Interpret percents as *x/100*, calculate percentages of whole numbers, generate equivalent decimals and percentages.
- Compare and order common fractions, decimals, and percents

- Division with multi-digit divisors (with and without decimals): Write remainders as whole numbers and express remainder amounts as fractions or decimals that are part of the quotient. (13/4 = 3 R1 = 3 ¼ = 3.25)
- Generate prime factors of numbers 1-100, and use exponents to show multiples of a factor

Geometry and Measurement
- Know and use formulas for perimeter, area, volume (cubic vs. squared units, concrete and representational), and circumference
- Area: lxw, sq.units, triangles
- Volume: Cubic vs. squared unit; concrete and representational
- Measure, identify, and draw common geometric shapes
- Perimeter: Area of polygons: concrete representational
- Apply knowledge that interior angles of a triangle add to 180° and interior angles of a quadrilateral add to 360° in order to solve problems.

Algebra and Functions
- Interpret simple algebraic expressions, variables
- Identify and graph ordered pairs in the four quadrants of the coordinate plane
- Use function tables (T- table, 3-table) to identify and continue patterns, graph constant growth patterns
- Fluency with equivalence signs: =, <, >,…..
- Fill in the blank, symbol, or letter with operational sign or number
- Solve equations with variables
- Compute with positive and negative integers

Statistics, Data Analysis, and Probability
- Understand the concepts of mean, median, and mode
- Organize, display, and interpret data on line and circle graphs
- Use fractions and percentages to compare data sets

Student work habits
- Use and organize a math journal
- Be on task and productive
- use multiple problem solving strategies
- work independently on investigation/practices
- Work constructively in small cooperative groups
- persevere through a task
- Articulate math thinking orally and in writing.
- Complete class work and homework in a timely fashion
- Produce math papers that are annotated correctly, legible, and well-organized
- Show thinking and work on written papers (with numbers, pictures, words)
- organize materials during class (manipulatives, paper, books)

- listen attentively during demonstration
- copy accurately off of board or text

Course I Essentials:

Number Sense

- Compare and order positive and negative fractions, decimals, percents and mixed numbers on a number line.
- Use ratios to show the relative size of two quantities and to solve problems with cross-multiplication
- Solve addition, subtraction, multiplication, and division problems with positive and negative integers, fractions and decimals
- Write and solve one-step linear equations with one variable
- **Scientific notation will be taught in grades 6-8 science

Geometry and Measurement

- Use complementary and supplementary angles and the sum of angles of a triangle to solve problems involving an unknown angle.
- Understand perimeter and how to calculate the perimeter of polygons and circles (diameter:circumference ratio; perimeter generalizations ($2lw$, etc.))
- Calculate area of polygons and circles

Algebra and Functions

- Write and solve one step equations with one variable
- Demonstrate understanding that rate is a ratio comparing two quantities of different units
- Understand the difference between operational signs (+,-,x,÷) and relational signs ($=,<,>,\leq,\geq,\neq,\approx$)

Statistics, Data Analysis, and Probability

- Design and conduct surveys; utilize measures of central tendency in data interpretation

Student work habits

- Use and organize a math journal
- Be on task and productive
- Use multiple problem solving strategies
- Work independently on investigation/practices
- Persevere through a task
- Verbally articulate math thinking.
- Complete class work and homework in a timely fashion
- Produce math papers that are legible and well-organized
- Show thinking and work on written papers (with numbers, pictures, words)
- Organize materials during class (manipulatives, paper, books)

- Listen attentively during demonstration
- Copy accurately off of board or text
- Interpret directions and persevere in multiple-step problem solving (e.g.: MARS tasks)
- Organize materials (books, journal, calculator, pencil, protractor, compass, homework papers) and assignment tracking routines.

Course II Essentials:

Number Sense

- Review use of greatest common factor and least common multiple in operations over rational numbers
- Demonstrate computational fluency in 4 basic operations over set of rational numbers
- Know and use order of operations to simplify a variety of expressions (PEMDAS)
- Use reciprocals, and commutative, associative, and distributive properties to solve multi-step equations with 1 unknown.
- Understand and use exponent rules, squares, and square roots.
- Gain fluency in fraction/decimal/percent conversions.
- Understand base 10 exponents as an expression of base 10 place value. (10^2=100; 10^{-2}=1/100)
- **scientific notation will be taught in grades 6-8 science

Geometry and Measurement

- Calculate volume of prisms and cylinders. Students will experience the concept of volume in concrete, representational, and abstract contexts.

Algebra and Functions

- Understand and use distributive, commutative, and associative properties
- Locate ordered pairs in 4 quadrants of the Cartesian coordinate system
- Use t-tables to generalize linear growth with an explicit function, and graph it on the coordinate plane.
- Represent linear growth with algebraic functions, concrete models or pictures, t-tables, language, and graphs.

Statistics, Data Analysis, and Probability

- Analyze data sets: range, median, mean, line plot, box and whiskers plot

Student work habits

- Use and organize a math journal to take useful notes in math class and to inform practice work.
- Be on task and productive.
- Use multiple problem solving strategies.
- Work independently on investigation/practices.

- Work constructively in small cooperative groups.
- Persevere through a task.
- Articulate math thinking orally and in writing.
- Complete class work and homework in a timely fashion.
- Produce math papers that are legible and well-organized.
- Listen attentively during demonstration.
- Copy accurately off of board or text.
- Interpret directions and persevere in multiple-step problem solving (e.g.: MARS tasks).
- Use calculator basic skills through exponents and square roots.
- Monitor own understanding and ask questions when it is not present.
- Organize materials (books, journal, calculator, pencil, protractor, compass, homework papers) and assignment tracking routines.

Course III Essentials:

Algebra I, Linear Algebra:

- Solve equations in one variable
    - one transformation: $5x + 4 = 24$
    - multiple transformations: $x + 4 = 2x - 6$
- Write and solve proportions in one variable: $x/4 = 9/12$
- Solve inequalities in one variable: $2(x - 3) > x + 2$
    - Represent solutions on the numberline
- Evaluate an expression for a given value of x:
    - Given $x = -9$, evaluate $2/3x - 2x^2 + 5$
- Understand and apply squares, roots, and exponent rules.
- Understand and solve problems using absolute value: $|-4| = x + 3$
- Graph points on a coordinate grid in all quadrants.
- Solve equations in two variables: $y = 4x + 5$
    - Show solutions in multiple representations, including t-table, graph, and function
- Graph functions using:
    - intercept-slope form
    - x and y intercepts
- Determine functions given:
    - two points
    - one point and the slope
- Understand and apply slope to problems involving rate
- Understand the concepts of parallel and perpendicular lines, and how their slopes are related.
- Solve systems of equations by graph, substitution, and elimination.

Student work habits

- Use and organize a math journal to take useful notes in math class and to inform practice work.

- Be on task and productive.
- Use multiple problem solving strategies.
- Work independently on investigation/practices.
- Work constructively in small cooperative groups.
- Persevere through a task.
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Course IV Essentials:

Algebra I, Non-Linear Algebra:

Non-linear Algebra

- Distingush between linear and non-linear patterns of growth and decay.
- Develop quadratic functions from collected data.
- Graph quadratic functions on the coordinate plane.
- Understand the effects of changing variables on the graph of a quadratic equation.
- Factor quadratic equations to roots, and correlate roots to x intercepts of a graph.
- Determine a quadratic function given:
    - three points on a graph
    - the vertex and x intercepts of a graph
- Solve quadratic equations by "completion of the square."
- Solve quadratic equations by the "quadratic formula."
- Identify the integral and derivative of the graph of a quadratic equation, and apply them in the solution to problems involving change in rate.
- Perform arithmetic operations on polynomial expressions.

Student work habits

- Use and organize a math journal to take useful notes in math class and to inform practice work.
- Be on task and productive.
- Use multiple problem solving strategies.
- Work independently on investigation/practices.

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- Work constructively in small cooperative groups.
- Persevere through a task.
- Articulate math thinking orally and in writing.
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Mathematics Vocabulary

Language is a very powerful tool and should be used to foster the learning of mathematics. Communicating about mathematical ideas is a way for students to articulate, clarify, organize, and consolidate their thinking. Communication makes mathematical thinking observable and thereforefacilitates further development of that thought. It encourages students to reflect on their own knowledge and their own ways of solving problems. Communication can consist not only of conversations between student and teacher or one student and another student but also of students listening to a number of peers and joining group discussions in order to clarify, question, and extend conjectures. The discourse should not be a goal in itself but rather should be focused on making sense of mathematical ideas and using them effectively in modeling solving problems.

From Principles and Standards for School Mathematics, **NCTM**

Each grade level vocabulary list is inclusive of all previous grade levels and should be utilized in context. This is not a finite vocabulary list but rather an ongoing, expanding file.

Course I-II Math Vocabulary

Number & Operations

Absolute Value
Commutative Property (of Addition and Multiplication)
Associative Property
Distributive Property
Identity Property (of Addition and Multiplication)
Equation
Inequality
Expression (numeric and algebraic)
Factor
Common Factor
Greatest Common Factor
Multiple
Common Multiples
Least Common Multiples
Product
Quotient
Sum
Difference
Operation
Inverse Operation
Reciprocal
Order of Operations
Estimate
Prime Number
Composite Number
Irrational Number
Rational Number
Repeating Decimal
Terminating Decimal
Prime Factorization
Factor Tree
Exponent
Power
Base
Square Root
Evaluate
Simplify
Integer
Variable

Fraction, Decimal, Percents

Fraction
Numerator
Denominator
Proper Fraction
Improper Fraction
Mixed Number
Equivalent Fraction
Decimal
Percent

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Simplest Form

Geometry & Measurement

Angle
Right Angle
Acute Angle
Obtuse Angle
Straight Angle
Degrees
Vertex
Ray
Coordinate Plane
Quadrant
X-axis
Y-axis
Horizontal
Vertical
Ordered Pair
Coordinates
Origin
Circle
Radius/Radii
Protractor
Diameter
Circumference
Pi
Polygon
Regular Polygon
Triangle
Equilateral Triangle
Isosceles Triangle
Scalene Triangle
Acute Triangle
Right Triangle
Obtuse Triangle
Quadrilateral
Square
Rectangle
Rhombus
Parallelogram
Trapezoid
Pentagon
Hexagon
Octagon
Decagon
Area
Dimensions
Squared
Volume
Cubic Unit
Prism
Cube
Formulas
Perimeter
Vertices
Faces
Edges
Congruent
Conversion
Meter/Liter/Gram
Mile/Yard/Foot/Inch
Gallon/Quart/Pint/Cup/Oz.
Dozen
Milli
Centi
Kilo
Pythagorean Theorum

Probability and Statistics

Mean
Median
Mode
Range
Minimum
Maximum
Outlier
Probability
Experimental Probability
Theoretical Probability
Independent Events
Dependent Events
Complement of Probability
Circle Graph
Bar Graph
Line Graph
Line Plot
Measure of Central Tendency
Stem and Leaf Plot
Venn Diagram
Sample Space
Organized List
Tree Diagram

Course III-IV Math Vocabulary

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Linear and Non-Linear Algebra

absolute value
axis
binomial
coefficient
colinear
combination
constant
coordinate plane
difference
differential
directrix
domain
elimination
equation
explicit function
exponent
expression
factored form
focus
function
inequality
integer
integral
intercept
inverse
like terms
linear function
monomial
non-linear function
parabola
perpendicular
polynomial
prime
product
proportionate
quadratic function
quadrant
range
rate
reciprocal
root
quotient
scale
similar
slope
substitution
sum
symmetry
trinomial
t-table
unike terms
variable
x intercept
y intercept
zeroth figure